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# The Analysis of the Dynamics of the Russian Economy Using the Output Gap Indicator

**A. V. Zubarev and P. V. Trunin**

*Center for the Study of Central Banks of the Russian Presidential Academy of National Economy and Public Administration (RANEPA), Moscow, 119571 Russia*

*e-mail: texxik@gmail.com*

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**Abstract**—In this paper we estimate the indicator *output gap* for the Russian economy in 2000–2015 using univariate and multivariate versions of the Hodrick–Prescott filter and the Kalman filter for the model of unobserved components (taking into account the Phillips curve). The calculation results show a slowdown of potential output after 2014.

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## ON THE METHODS OF ASSESSING THE OUTPUT GAP

In the analysis of the economic dynamics, the value of the so-called output gap, which is calculated based on estimates of potential output, is an important indicator. As a rule, in the economic literature, *output gap* refers to the percentage deviation of actual output from a certain expected potential level of output that corresponds to the natural unemployment rate. Output gap is considered to be an indicator of the imbalance between aggregate demand and aggregate supply and, hence, the presence of inflationary (deflationary) pressures in the economy.

This indicator can be used by monetary-control authorities in assessing the adoption of the necessary expansionary or, conversely, deterrent measures. Central banks of several countries are developing monetary policy in accordance with the evaluation of the output-gap indicator. Examples include the Bank of Canada [1], the Reserve Bank of New Zealand [2], the Central Bank of Brazil [3, 4], etc.

According to the basic assumptions of Keynesian theory, the economic system is almost never in equilibrium; the value of actual output is usually different from the equilibrium value, which corresponds to the natural unemployment rate, which is considered to be potential. A positive output gap is an indicator of the deviation of aggregate demand from its equilibrium level, which leads to the need for countercyclical macroeconomic policy. This is why it is important for central banks and other authorities in charge of economic policy to pay close attention to estimates of the value of this indicator.

Among economists, there is no single opinion about the optimal method of output gap estimation. Therefore, in order to obtain more reliable results, it is natural to use several different methods. Three groups of basic approaches can be distinguished. The first is univariate statistical procedures that are based on statistical properties of the series and are filtering or smoothing. The Hodrick–Prescott filter [5] is the most prominent in this group. A significant disadvantage of this filter, in addition to the absence of any theoretical foundations of the result obtained, is the problem of the loss of symmetry of the filter structure at the extreme points of the time series (see [6]).

When using the second group of approaches, univariate procedures can be extended to multivariate (called semi-structural) by adding the equations that take into account theoretical assumptions about the relationship of the potential and actual output levels with other macroeconomic indicators. These relationships are based on theoretical concepts and identified empirical regularities linking the actual and potential output with the levels of inflation, unemployment and other macroeconomic indicators (e.g., the Phillips curve). Paper [1] was one of the first to implement this approach; it was also used by the Reserve Bank of New Zealand [2]. However, it was shown (see [7]) that the appearance of new data on the output level and other indicators when using the multivariate Hodrick–Prescott filter may lead to the same major revision of potential output estimates, as well as when using the univariate Hodrick–Prescott filter. Semi-structural models also include unobserved components models (output gap assessments are obtained using the Kalman filter). This methodology is used, for example, in the Central Bank of Brazil [3, 4].

The third group includes so-called structural approaches, which include methods based on apply-

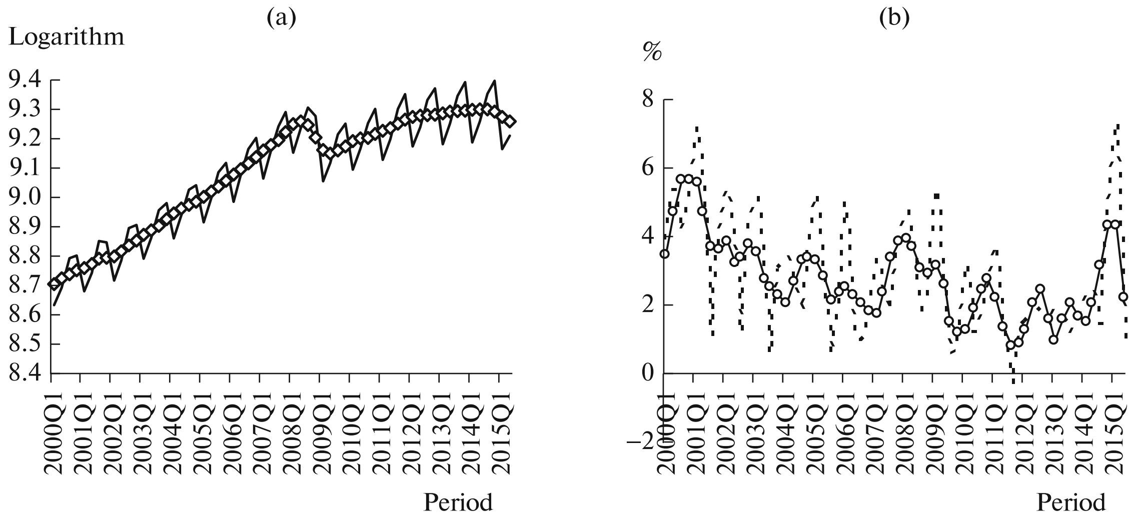


**Fig. 1.** Series of (a) the GDP index in 2000–2015 and (b) inflation of Russia: **—** output logarithm; –◇– seasonally smoothed output; **----** inflation; –○– seasonally smoothed inflation.

ing the production function [8], calculations of potential output using the dynamic stochastic general equilibrium models (DSGE models) and the structural vector autoregression model. Russian experience in structuring with decomposition of GDP growth rates in the structural and market components is presented in [9].

As a rule, DSGE models are based on a fairly rigorous theoretical justification. Furthermore, the results largely depend on the assumptions innate in the model, and the construction of the model itself is a very time consuming task. An estimate of the potential output based on the production function is used, e.g., at the Bank of Japan [10, 11], based on theoretical ideas about the factors and the nature of their influence on the level of output in the long run. A disadvantage of this approach is a complicated process of defining the natural volume for the use of various production factors. Thus, data about the normal level of labor force employment are often prepared using univariate filtering or smoothing procedures, where there is a problem of displacement estimates at the extremes of the sample. Moreover, the results depend largely on the type and calibration of the production function. Consider the results of applying the described uni- and multivariate statistical procedures to analyzing the time series of the Russian GDP.

## DATA DESCRIPTION

The main variable that was used in all calculations is the quarterly series of the Russian GDP index in 2008 prices, Rosstat published this series from the first quarter of 1995 to the fourth quarter of 2011; however, the International Monetary Fund published this series until the end of 2014. To obtain a longer series, we have extended it to the 2nd quarter of 2015 using the figures of GDP growth rate obtained from a series of real GDP in 2011 prices published by Rosstat. We have taken data from 2000, as the Russian economy underwent significant structural changes in the preceding period. The plotted chart of the series of GDP is shown in Fig. 1a, which shows that the dynamics of the Russian GDP has a characteristic seasonal component. In connection with this, a reasonable step is the seasonal adjustment of the series. One of the most common methods in this field is smoothing using the X-13 procedure (Fig. 1a; authors' calculation [12]).

In addition to the output, in our calculations, we used the quarterly series of inflation derived from the quarterly series of the consumer price index (CPI) published by Rosstat. This series also has a strong seasonal component (Fig. 1b; authors' calculation [12]). The X-13 procedure was also used for seasonal adjustment.

## ESTIMATES OF OUTPUT GAP

To determine the output gap, we used uni- and multivariate statistical procedures, as well as the semi-structural models. The univariate Hodrick–Prescott filter is the procedure for series smoothing by minimizing the following functional:

$$\sum_{t=1}^{T}(y_t - \tau_t^y)^2 + \lambda\sum_{t=2}^{T-1}[(\tau_{t+1}^y - \tau_t^y) - (\tau_t^y - \tau_{t-1}^y)]^2, \quad (1)$$

where $y_t$ is the actual output, $\tau_t^y$ is the potential output, $\lambda$ is the degree of smoothness of the series obtained as a result of this statistical procedure and $T$

is the range. For the quarterly data, we used the value $\lambda = 1600$.

The problem of a symmetric filter shift at the terminal points of the series has been solved by introducing additional members in the functional that limit excessive volatility of the output gap at the ends of the sample. The new functionality is as follows:

$$\sum_{t=1}^{T}(y_t - \tau_t^y)^2 + \lambda\sum_{t=2}^{T-1}[(\tau_{t+1}^y - \tau_t^y) - (\tau_t^y - \tau_{t-1}^y)]^2 + \omega((y_T - \tau_T^y)^2 + (y_{T-1} - \tau_{T-1}^y)^2), \quad (2)$$

where two new members change (increase) weights (defined by the parameter $\omega = 2$) deviations at the last two observations in the functional.

The univariate Hodrick–Prescott filter is also upgraded to the multivariate filter by the Phillips curve equation, which represents the dependence of the observed inflation on the past values of inflation (characterizes adaptive expectations), the future expected value of inflation (characterizes rational expectations), and the output gap. Estimates of the Phillips curve equation for the Russian economy are taken from [13]. This method is used, e.g., at the Reserve Bank of New Zealand [2]. As a result, the multivariate Hodrick–Prescott filter represents the maximization of the following functional:

$$\sum_{t=1}^{T}(y_t - \tau_t^y)^2 + \lambda\sum_{t=2}^{T-1}[(\tau_{t+1}^y - \tau_t^y) - (\tau_t^y - \tau_{t-1}^y)]^2 + \sum_{t=1}^{T}\lambda_t^{\pi}(\varepsilon_t^{\pi})^2, \quad (3)$$

where $\varepsilon_t^{\pi}$ represents excesses that result from the estimate of the Phillips curve equation. We set the parameter $\lambda_t^{\pi}$ equal to 20.[1]

The Phillips curve equation is as follows:

$$\pi_t = c + \beta_1\pi_{t-1} + \beta_2\pi_{t+1} + \beta_3 z_{t-1} + \varepsilon_t, \quad (4)$$

where $\pi_t$ is inflation and $z_t$ is the output gap.

The semi-structural unobserved components model gives some idea of the processes that determine the expansion of the observed GDP in the economy on the potential output and output gap. We assume that the potential GDP is a random walk with a drift and the drift itself is also generated by a random walk, i.e., the drift is not a constant, but rather represents a stochastic process. It is assumed that the output gap is defined by a stationary autoregressive process of the second order. The specification of the model is as follows:

$$y_t = y_t^p + z_t, \quad (5)$$

$$y_t^p = \mu_{t-1} + y_{t-1}^p + \varepsilon_t^p, \quad (6)$$

$$\mu_t = \mu_{t-1} + \varepsilon_t^{\mu}, \quad (7)$$

$$z_t = \alpha_1^* z_{t-1} + \alpha_2^* z_{t-2} + \varepsilon_t^z, \quad (8)$$

where the first equation is the expansion of real output $y_t$ to potential output $y_t^p$ and output gap $z_t$; $\mu_t$ corresponds to the growth rate of potential output; and $\varepsilon_t^{\mu}$, $\varepsilon_t^z$, $\varepsilon_t^p$ are the errors (assumed to be normal, independent, and identically distributed).

As in the case of the Hodrick–Prescott filter, one can add the Phillips curve equation to the unobserved components model, which makes it possible to take into account the inflationary pressure that occurs upon a deviation of the actual output from the potential. For this purpose, as before, one should add the equation to the system described above:

$$\pi_t = \beta_1\pi_{t-1} + \beta_2\pi_{t+1} + \beta_3 z_t + \varepsilon_t^{\pi}, \quad (9)$$

where $z_t$ is the output gap, $\pi_t$ is inflation, and $\varepsilon_t^{\pi}$ represents errors.

Unobserved components are extracted from the model described above by means of the Kalman filter. In this case, the equations of state (all the equations except that of output expansion on the potential output and output gap) are estimated using, not the standard method of maximum likelihood, but Bayesian estimators, as in complex models (requiring the estimation of a large number of parameters) numerical methods used to obtain maximum likelihood estimates work unstably; the reason for this may be the inaccurate choice of the initial conditions.

As the average values of a priori distributions of the autoregression equation coefficients for the output gap, we used the point estimates from regressions with output gap and potential output, which are obtained using the basic version of the Hodrick–Prescott filter. Also, from these regressions, we took the average values for the a priori distribution of errors of the output gap equations and potential output. As the average values of a priori distributions for the inflation equation, we used the point estimates of one of the specifications set out in [13]. Tables 1 and 2 (authors' calculations) show the characteristics of a priori and a posteriori distributions of the parameters for the two versions of the model of unobserved components. As a priori distributions of the coefficients, we used normal distributions; for the error variance, we used inverse gamma distribution. According to the tables, the point estimates of the coefficients in a posteriori distributions are only slightly different from a priori (a priori esti-

[1] There is no theoretical justification for the choice of the value of this parameter, but the one that we used fully stays within the range of values in similar studies.

**Table 1.** Characteristics of a priori and a posteriori distributions of the model of unobserved components

| Parameter | A priori estimate | A posteriori estimate | Confidence interval | | A priori distribution | A priori variance |
|---|---|---|---|---|---|---|
| $\alpha_1$ | 1.38 | 1.4328 | 1.3175 | 1.5648 | Normal | 0.1 |
| $\alpha_2$ | −0.59 | −0.5516 | −0.6525 | −0.458 | Normal | 0.1 |
| $\mathrm{Var}(\varepsilon_t^p)$ | 0.001 | 0.002 | 0.0004 | 0.0033 | Inverse gamma distribution | Undetermined |
| $\mathrm{Var}(\varepsilon_t^p)$ | 0.002 | 0.0011 | 0.0005 | 0.0016 | Inverse gamma distribution | Undetermined |
| $\mathrm{Var}(\varepsilon_t^p)$ | 0.06 | 0.0133 | 0.0116 | 0.015 | Inverse gamma distribution | Undetermined |

**Table 2.** Characteristics of a priori and a posteriori distributions of the model of unobserved components with the added Phillips curve

| Parameter | A priori estimate | A posteriori estimate | Confidence interval | | A priori distribution | A priori variance |
|---|---|---|---|---|---|---|
| $\alpha_1$ | 1.38 | 1.4084 | 1.3094 | 1.5117 | Normal | 0.1 |
| $\alpha_2$ | −0.59 | −0.5591 | −0.6599 | −0.4628 | Normal | 0.1 |
| $\gamma_1$ | 0.5 | 0.5219 | 0.45 | 0.594 | Normal | 0.05 |
| $\gamma_2$ | 0.29 | 0.3011 | 0.2241 | 0.3875 | Normal | 0.05 |
| $\gamma_3$ | 25 | 24.5192 | 20.8948 | 27.678 | Normal | 2 |
| $\mathrm{Var}(\varepsilon_t^p)$ | 0.002 | 0.0023 | 0.0011 | 0.0036 | Inverse gamma distribution | Undetermined |
| $\mathrm{Var}(\varepsilon_t^p)$ | 0.002 | 0.0014 | 0.0005 | 0.0024 | Inverse gamma distribution | Undetermined |
| $\mathrm{Var}(\varepsilon_t^p)$ | 0.06 | 0.0128 | 0.0111 | 0.0145 | Inverse gamma distribution | Undetermined |
| $\mathrm{Var}(\varepsilon_t^p)$ | 0.005 | 0.0041 | 0.0015 | 0.0069 | Inverse gamma distribution | Undetermined |

mates of the coefficients lie within a posteriori confidence sets), which indicates the adequacy of a priori judgments.

Figure 2 (authors' calculations) provides the graphs of the output gap estimates for all of the considered models. First and foremost, it is worth noting a similar nature of the dynamics of output gap in different models; with the exception of the last two sample points, all of the estimates of the output gap are within a bandwidth of 2% (directly in terms of the value of output gap).

In [13], 12 different versions of the Phillips curve were evaluated. For each of version of the curve, we built a series of the output gap. Figure 2[2] shows a band (of minimum width) that contains all twelve series of the output gap. The lower limit of the band differs from the upper limit by no more than 1%; furthermore, the estimate of the output gap using the basic version of the Hodrick–Prescott filter is nearly identical to the upper limit of this band.

Thus, although the addition of the Phillips curve in the Hodrick–Prescott filter leads to some changes in the quantitative estimates, the dynamics of the estimated output gap is not qualitatively changed.

The Hodrick–Prescott filter with the correction of weights of terminal points in the functional is an attempt to solve the problem of the displacement of this filter at the edges of the sample due to the loss of symmetry. It should be understood that the choice of weighting factor value $\omega$ is devoid of theoretical assumptions. Nevertheless, in a certain sense, this corrected version of the Hodrick–Prescott filter, which limits excessively sharp jumps of the output gap at the extreme points, gives some information about its possible shift. Thus, Fig. 2 shows that the adjusted filter shows a higher output gap in comparison with the basic version of the filter for the past ten or twelve quarters. Estimates of output gap for the last two quarters turned out to be approximately 1.5% higher.

[2] In Fig. 2 HP is the basic version of the Hodrick–Prescott filter; Kalman and Kalman PC correspond to the specification of the model of unobserved components without the Phillips curve and with it, accordingly; Min and Max correspond to the minimum and maximum values of the multivariate Hodrick–Prescott filter with the added Phillips curve equation; HP Corr is the multivariate Hodrick–Prescott filter with the correction of weights for the last two sample points.

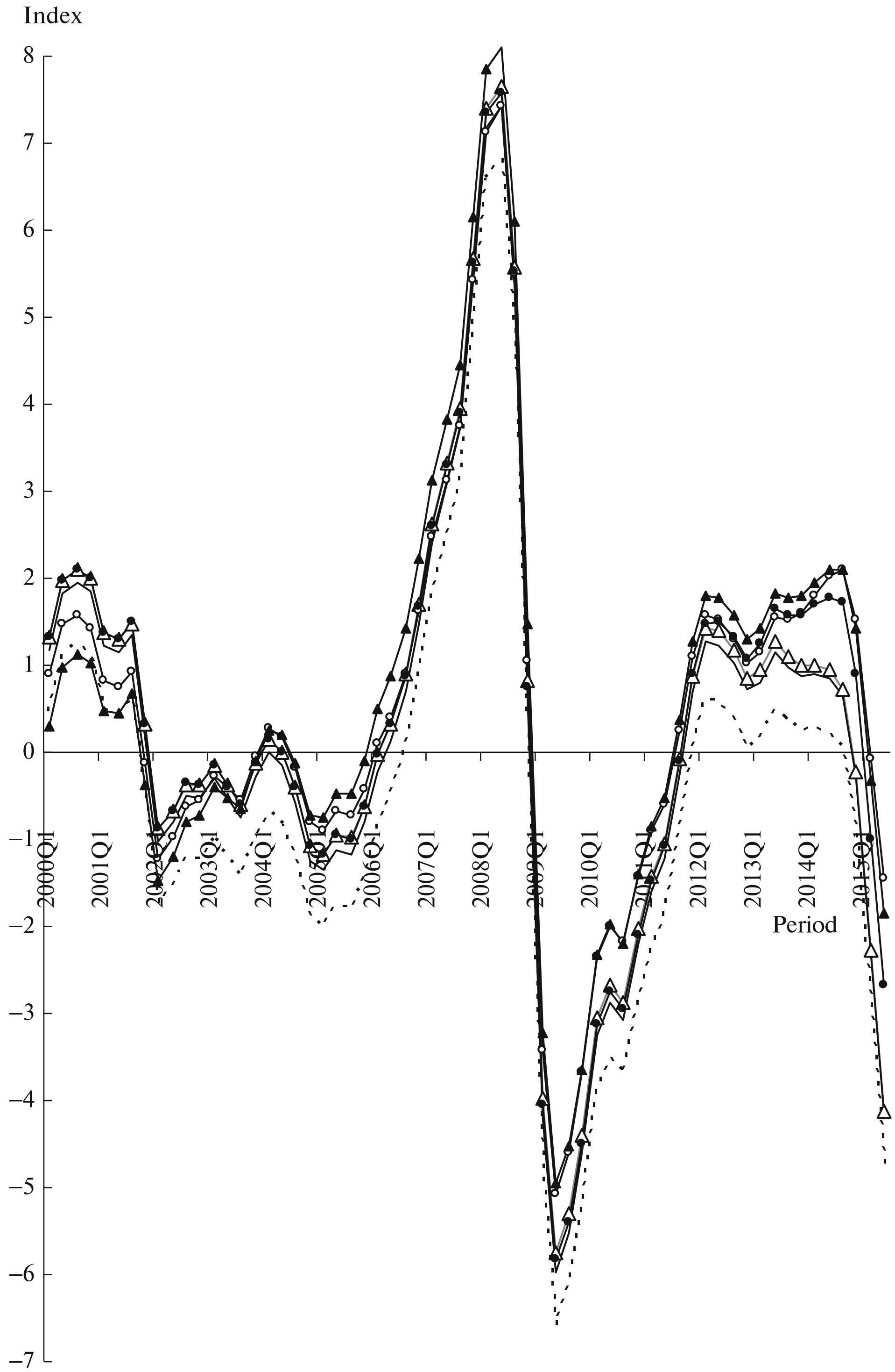


**Fig. 2.** Output gap in 2000–2015: — Max; ---- Min; –○– Kalman; –●– HP Corrected; –▲– Kalman PC; –△– HP.

The series of the output gap obtained using the Kalman filter demonstrate similar estimates and show higher values of output gap in 2015. At the same time, it is clear that the addition of the Phillips curve to the unobserved component model also does not introduce significant changes to the result. It consists of estimates obtained using the Kalman filter that, in our opinion, are the most reliable, as the used models of unobserved components take into account some ideas of the structure of unobserved series. The similarity of its result to the result of the adjusted Hodrick–Prescott filter also counts in favor of the models of unobserved components.

## ANALYSIS OF THE RESULTS

In 2000 and 2001, in the initial phase of the recovery growth of the Russian economy after the crisis of 1998–1999, there was a positive output gap – up to 1.5–2% (see Fig. 2). In other words, about this much the observed GDP was above its potential level. In the following period of 2002–2006, which conformed to the economic growth phase with increasing prices for major Russian export commodities, the output gap was mainly in the region of zero, i.e., observed output almost fully conformed to a potential one.

Since 2006, the overheating of the economy began to be observed and, according to various estimates, the output gap gradually reached a peak at the level of 7–8% in 2008. This significant positive output gap indicates that, during this period labor, capital and other factors of production were used in volumes much greater than the natural rate of their use. Note that, at this stage, the economy was characterized by a relatively loose monetary and fiscal policy, which, together with the influx of currency from abroad as a result of rising prices for energy carriers and private-sector capital inflows, exerted increased inflationary pressures on the economy of the Russian Federation. The observed decline in GDP during the crisis by more than 10% was associated with the deflation of bubbles in some markets that have arisen precisely because of the significant overheating of the economy. This is confirmed, e.g., by the results of [14], in which the authors based on an analysis of the constructed models of vector autoregression conclude that the policy of the Bank of Russia in 2008–2009 was procyclical in nature. They also found that the shocks of monetary policy explain up to a 1.5% deviation in the output from the trend. Estimates obtained by us as a result of this study show that, in 2009, the output gap fell below 5%.

Since 2011, after the economic recovery output gap was again only slightly (1–2%) higher than the zero point. However, in 2014, with the beginning of a new phase of the crisis in the Russian economy, associated with the fall of oil prices in the world and the introduction of economic sanctions against Russia, output gap began to decline. According to the most reliable, in our opinion, estimates obtained using the Kalman filter, output gap was approximately at the level of –2% in the 2nd quarter of 2015.

It is important that the less significant (compared to the basic version of the Hodrick–Prescott filter) negative output gap in 2015 was due to a significant reduction in potential output in the Russian economy. One of the channels of potential reduction output is the reduction of the equilibrium values of the employed capital level, which happened due to the weakening of the national currency and the rise in the price of investment goods due to the negative shock: financial sanctions and terms of trade. In addition, as a result of the crisis processes in the Russian economy, the uncertainty of prospects for further development increased sharply, which had an extremely negative impact on the investment process. Another important factor in reducing the potential output may be the fall in total factor productivity as a result of restrictions on borrowing capital and technology; not only the devalued national currency, but also the economic sanctions limit the ability to attract investors from abroad.

## CONCLUSIONS

The results indicate that the decline in GDP in 2015 was caused not so much by the negative output gap as a decrease in potential output. Thus, for the first half of 2015, there was a drop in the observed output (series, cleared of seasonality) by 3.5%; furthermore, according to the assessment obtained using the Kalman filter, the potential output decreased by 3.3%. This situation can be interpreted as a result of the structural shift that took place in the economy, which is a transition to a low long-term rate of economic growth after the relatively high growth rates that preceded the global financial crisis of 2008–2009. We conclude that a slight negative output gap at a level of 2% does not require the use of significant measures to increase the aggregate demand in the economy, since they cannot affect the long-term economic growth. At the same time, these measures can cause inflationary pressures and capital outflows.

In conclusion, we emphasize that any assessments of output gap essentially depend on assumptions laid in the model and therefore are subjective. These assessments should not be the only tool in making economic policy decisions, but of course they should be take into account by the authorities of monetary control.

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